

$$\textcircled{1} \left[\frac{17}{x^2+1} - \frac{22}{(x^2+1)^2} = 3 \right] (x^2+1)^2$$

$$17(x^2+1) - 22 = 3(x^2+1)^2$$

$$17x^2 + 17 - 22 = 3(x^4 + 2x^2 + 1)$$

$$17x^2 - 5 = 3x^4 + 6x^2 + 3$$

$$3x^4 - 11x^2 + 8 = 0$$

$$\text{let } y = x^2$$

$$3y^2 - 11y + 8 = 0$$

$$(3y - 8)(y - 1) = 0$$

$$y = \frac{8}{3} \quad | \quad y = 1$$

$$x^2 = \frac{8}{3} \quad x^2 = 1$$

$$x = \pm \sqrt{\frac{8}{3}} \quad x = \pm 1$$

$$x = \pm \frac{2\sqrt{6}}{3}$$

$$x = \pm \frac{2\sqrt{6}}{3} \quad \left\{ \pm \frac{2\sqrt{6}}{3}, \pm 1 \right\}$$

$$\textcircled{2} 1 + 3(x^2-1)^{-1} = 28(x^2-1)^{-2}$$

$$\text{let } y = (x^2-1)^{-1}$$

$$1 + 3y = 28y^2$$

$$28y^2 - 3y - 1 = 0$$

$$(7y + 1)(4y - 1) = 0$$

$$y = -\frac{1}{7} \quad | \quad y = \frac{1}{4}$$

$$(x^2-1)^{-1} = -\frac{1}{7} \quad (x^2-1)^{-1} = \frac{1}{4}$$

$$x^2-1 = -7 \quad x^2-1 = 4$$

$$x^2 = -6 \quad x^2 = 5$$

$$x = \pm i\sqrt{6} \quad x = \pm \sqrt{5}$$

$$\{ \pm i\sqrt{6}, \pm \sqrt{5} \}$$

$$\textcircled{4} 2(1+2\sqrt{x})^2 - (1+2\sqrt{x}) = 21$$

$$\text{let } y = 1+2\sqrt{x}$$

$$2y^2 - y = 21$$

$$2y^2 - y - 21 = 0$$

$$(2y - 7)(y + 3) = 0$$

$$y = \frac{7}{2} \quad | \quad y = -3$$

$$1+2\sqrt{x} = \frac{7}{2}$$

$$2\sqrt{x} = \frac{5}{2}$$

$$\sqrt{x} = \frac{5}{4}$$

$$x = \frac{25}{16}$$

$$1+2\sqrt{x} = -3$$

$$2\sqrt{x} = -4$$

$$\sqrt{x} = -2$$

$$\left\{ \frac{25}{16} \right\}$$

$$\textcircled{3} x^3 - 13x^{3/2} + 40 = 0$$

$$\text{let } y = x^{3/2}$$

$$y^2 - 13y + 40 = 0$$

$$(y - 5)(y - 8) = 0$$

$$y = 5 \quad | \quad y = 8$$

$$x^{3/2} = 5 \quad x^{3/2} = 8$$

$$x^3 = 25 \quad x^3 = 64$$

$$x = \sqrt[3]{25} \quad x = 4$$

$$\{ 4, \sqrt[3]{25} \}$$

$$\textcircled{5} \sqrt{3x+4} - \sqrt{x+1} = 1$$

$$(\sqrt{3x+4})^2 = (\sqrt{x+1} + 1)^2$$

$$3x+4 = x+1 + 2\sqrt{x+1} + 1$$

$$2x+2 = 2\sqrt{x+1}$$

$$(x+1)^2 = (\sqrt{x+1})^2$$

$$x^2+2x+1 = x+1$$

$$x^2+x = 0$$

$$x(x+1) = 0$$

$$x = 0 \quad x = -1$$

$$\text{Check } x=0$$

$$\sqrt{4} - \sqrt{1} = 1$$

$$2 - 1 = 1$$

$$\text{Check } x=-1$$

$$\sqrt{-3+4} - \sqrt{-1+1} = 1$$

$$1 - 0 = 1 \checkmark$$

$$\{ 0, -1 \}$$

Unit 9 2.6 Worksheet - continued

$$\textcircled{6} (\sqrt{5x-1} + \sqrt{2-x}) = (\sqrt{8x+1})^2 \quad \textcircled{7} (\sqrt{2\sqrt{x+2}})^2 = (\sqrt{3x+2})^2$$

$$5x-1+2\sqrt{(2-x)(5x-1)}+2-x=8x+1$$

$$(2\sqrt{x+2})^2 = (3x+2)^2$$

$$2\sqrt{(2-x)(5x-1)} = 4x$$

$$4(7x+2) = 9x^2 + 12x + 4$$

$$\sqrt{(2-x)(5x-1)} = 2x$$

$$28x+8 = 9x^2 + 12x + 4$$

$$(2-x)(5x-1) = 4x^2$$

$$0 = 9x^2 - 16x - 4$$

$$10x - 2 - 5x^2 + x = 4x^2$$

$$0 = (9x+2)(x-2)$$

$$9x^2 - 11x + 2 = 0$$

$$x = -\frac{2}{9} \quad | \quad x = 2$$

$$(9x-2)(x-1)$$

$$3x+2 \geq 0 \quad 2\sqrt{x+2} \geq 0$$

$$x = \frac{2}{9} \quad | \quad x = 1$$

$$3x-2 \geq 0 \quad \sqrt{x+2} \geq 0$$

$$\left\{1, \frac{2}{9}\right\}$$

$$x \geq \frac{2}{3} \quad 7x+2 \geq 0$$

$$\left\{-\frac{2}{3}, -\frac{2}{7}\right\}$$

$$7x \geq -2$$

$$x \geq -\frac{2}{7}$$

$$\textcircled{8} (\sqrt{x} + 2)^2 = (\sqrt{4+7\sqrt{x}})^2$$

$$\textcircled{9} (2x-1)^{2/3} = x^{1/3}$$

$$\textcircled{10} x(2+x)^{-1/2} + (2+x)^{1/2} = 0$$

$$x + 4\sqrt{x} + 4 = 4 + 7\sqrt{x}$$

$$(2x-1)^2 = x$$

$$[x(2+x)^{-1/2}]^2 = [(2+x)^{1/2}]^2$$

$$x = 3\sqrt{x} \quad \text{check}$$

$$4x^2 - 4x + 1 = x$$

$$x^2(2+x)^{-1} = (2+x)$$

$$x^2 = 9x \quad \sqrt{x} + 2 \geq 0$$

$$4x^2 - 5x + 1 = 0$$

$$\frac{x^2}{2+x} = \frac{2+x}{1}$$

$$x^2 - 9x = 0 \quad \sqrt{x} \geq -2$$

$$(4x-1)(x-1) = 0$$

$$x^2 = (2+x)^2$$

$$x(x-9) = 0 \quad \sqrt{x} \geq 0$$

$$x = \frac{1}{4} \quad | \quad x = 1$$

$$x^2 = 4 + 4x + x^2$$

$$x = 0 \quad x = 9 \quad \sqrt{4+7\sqrt{x}} \geq 0$$

$$\left\{\frac{1}{4}, 1\right\}$$

$$0 = 4 + 4x$$

$$\{0, 9\}$$

$$4 + 7\sqrt{x} \geq 0$$

$$7\sqrt{x} \geq -4$$

$$\sqrt{x} \geq -\frac{4}{7}$$

$$\sqrt{x} \geq 0$$

$$x \geq 0$$

$$4x = 4$$

$$x = 1$$

$$\{-1\}$$

check

$$-1(2+1)^{-1/2} + (2-1)^{1/2} = 0$$

$$-1(1)^{-1/2} + (1)^{1/2} = 0$$

$$-1 + 1 = 0$$

$$0 = 0 \quad \checkmark$$